ME2 Computing- Coursework summary

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A) What physics are you trying to model and analyse? (Surely that is crazy!)

We are trying to analyse the heat transfer through a hot long rectangular rod when quenched in a cold fluid. To analyse, we are aiming to model the temperature distribution through the rod, as it changes in time. The rod is a composition of two different materials; in this case we chose brick and steel (AISI 1010) although it can be easily changed to fit any composition. Steel rods are commonly used in mechanical engineering and industrial applications, where quenching is a commonly used technique to make them stronger and harder. The rod is long enough that the heat transfer along the longest length can be neglected, so the heat transfer was modelled in two space dimensions and time. The square cross-sectional length of the rod is L (1m for the model shown here). The location and area of the secondary material was chosen to be a small rectangle, slightly offset from the centre, however, can be adjusted accordingly to model different problems. The rod is cooled for a total time of t seconds.

B) What PDE are you trying to solve? (write the PDE)

Assuming uniform density, uniform specific heat, no internal heat sources:

$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$ 2D Heat diffusion equation (source-free). (3rd order Parabolic PDE)

C) Boundary value and/or initial values for my specific problem: (be consistent with what you wrote in A)

The temperature distribution in the rod is described as $T_{x,y,t}$ where x and y are the coordinates in space, and t in time.

Initially, the rod is removed from a hot source, and it is at a uniform temperature of 900°C. This can be altered in the code.

$$T_{x,y,0} = 1173\ K$$

The boundaries at the edges of the rod are convective heat transfer boundary conditions. The 4 boundaries exposed to the fluid of temperature, T_f have the following boundary conditions:

Left edge: $-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = h(T_f - T_{0,y,t})$ Bottom edge: $-k\left.\frac{\partial T}{\partial y}\right|_{y=0} = h(T_f - T_{x,0,t})$

Right edge: $k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h(T_f - T_{L,y,t})$ Top edge: $k\left.\frac{\partial T}{\partial y}\right|_{y=L} = h(T_f - T_{x,L,t})$

D) What numerical method are you going to deploy and why?

We chose to use explicit methods because it is a fast and computationally efficient method. We used the finite difference method and the derivatives were calculated using central difference for our PDE. We also used forward difference and backward difference for our boundary conditions. These choices were made mainly to have an algorithm as simple as possible to make it easy to alter for different materials, shapes and sizes.

E) I am going to discretise my PDE as the following:

Discretised **space and time domains**:

N=number of nodes in x or y domain Where for a square grid, $\Delta x = \Delta y = L/(N-1)$, Δx = mesh grid

$$x_i = i \times \Delta x, \quad i = 0, 1, \ldots, N-1$$
$$y_j = j \times \Delta x, \quad j = 0, 1, \ldots, N-1$$

r=number of time nodes where, $\Delta t = t/(r-1)$

$$t_k = k\Delta t, \quad k = 0, 1, \ldots, r-1$$

Discretising **boundary conditions**, defining $g = \frac{k}{h\Delta x}$:

Left edge: (..difference)	$h\left(T_{0,j}^{k+1} - T_f\right) = k\left(\frac{T_{1,j}^{k} - T_{0,j}^{k+1}}{\Delta x}\right)$	$\Rightarrow T_{0,j}^{k+1} = \left(\frac{1}{1+g}\right)\left(T_f + gT_{1,j}^{k}\right)$
Right edge: (.. difference)	$h\left(T_f - T_{N-1,j}^{k+1}\right) = k\left(\frac{T_{N-1,j}^{k+1} - T_{N-2,j}^{k}}{\Delta x}\right)$	$\Rightarrow T_{N-1,j}^{k+1} = \left(\frac{1}{1+g}\right)\left(T_f + gT_{N-2,j}^{k}\right)$
Bottom edge: (.. difference)	$h\left(T_{i,0}^{k+1} - T_f\right) = k\left(\frac{T_{i,1}^{k} - T_{i,0}^{k+1}}{\Delta x}\right)$	$\Rightarrow T_{i,0}^{k+1} = \left(\frac{1}{1+g}\right)\left(T_f + gT_{i,1}^{k}\right)$
Top edge: (.. difference)	$h\left(T_f - T_{i,N-1}^{k+1}\right) = k\left(\frac{T_{i,N-1}^{k+1} - T_{i,N-2}^{k}}{\Delta x}\right)$	$\Rightarrow T_{i,N-1}^{k+1} = \left(\frac{1}{1+g}\right)\left(T_f + gT_{i,N-2}^{k}\right)$

E) I am going to discretise my PDE as the following (cont…)

Discretising **PDE** using explicit method and replacing partial derivatives using finite difference (central in space) (forward in time):

$$\frac{T_{i,j}^{k+1}-T_{i,j}^{k}}{\Delta t}=\alpha\left[\left(\frac{T_{i+1,j}^{k}-2T_{i,j}^{k}+T_{i-1,j}^{k}}{\Delta x^2}\right)+\left(\frac{T_{i,j+1}^{k}-2T_{i,j}^{k}+T_{i,j-1}^{k}}{\Delta x^2}\right)\right]$$

$$\Rightarrow T_{i,j}^{k+1}=T_{i,j}^{k}+\frac{\alpha\Delta t}{\Delta x^2}\left[\left(T_{i+1,j}^{k}-2T_{i,j}^{k}+T_{i-1,j}^{k}\right)+\left(T_{i,j+1}^{k}-2T_{i,j}^{k}+T_{i,j-1}^{k}\right)\right]$$

Defining $d=\frac{\alpha\Delta t}{\Delta x^2}$ and simplifying to obtain equation for all **internal nodes**:

$$\boxed{\Rightarrow T_{i,j}^{k+1}=T_{i,j}^{k}+d\left[T_{i+1,j}^{k}+T_{i-1,j}^{k}+T_{i,j+1}^{k}+T_{i,j-1}^{k}-4T_{i,j}^{k}\right]}$$

For two different materials:

$$d=d(x,y)=\begin{cases} d_{steel}, & 0.2\le x\le 0.8 \text{ and } 1.2\le y\le 0.8 \\ d_{brick}, & otherwise\end{cases}$$

$d_{steel}=\alpha_{steel}(\Delta t/(\Delta x^2))$ where, $\alpha_{steel}=1.88\times10^{-5}\ m^2/s$

$d_{brick}=\alpha_{brick}(\Delta t/(\Delta x^2))$ where, $\alpha_{brick}=5.2\times10^{-7}\ m^2/s$

F) Plot of results and comments (discuss how the results describe the physics and comment on any discrepancies or unexpected behaviours):

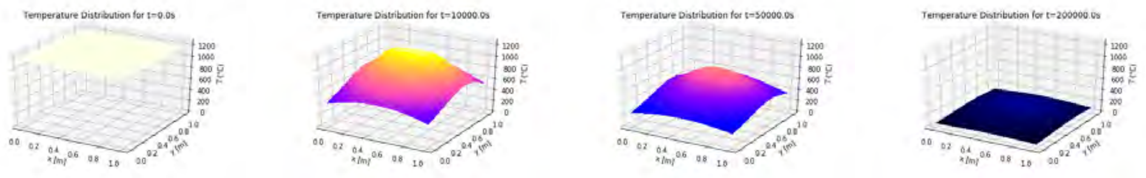

Figure 1. Figure of obtained temperature distributions at times: 0, 10000, 50000 and 200000 seconds.

The results shown in figure 1 show how the edges of the rod cool very quickly, and the core of the rod gradually reaches the same temperature over a larger amount of time. It also shows how the region of the beam with lower diffusivity (the brick) cools at a slower rate than the rest of the beam, remaining at higher temperatures for longer. After a period of 200,000 seconds (approximately 2 days), the entire rod reaches near T_f, the temperature of the fluid.

Although this value seems abnormally high, it is expected due to the large cross-sectional area of the shape (1x1m). While this problem could be applied to more realistic scales of smaller dimensions, the temperature distribution would not show noticeable differences at different locations (i.e. would yield a flatter surface).

It is also visible that there are no “sharp edges”; heat is transferred smoothly through the rod as expressed by the gradient in the colours and the smooth curve. Furthermore, the corners of the rod cool down the fastest. All of this is as we would expect from heat transfer theory.

G) Other remarks (limits of the model, convergence problems, possible alternative approaches, anything you find relevant and important to mention):

- The model can only be used for quite large cross sectioned rods. For too small rods, the temperature distribution is simply not visible as the heat transfer occurs too quickly. The model would still work, although it would not give much interesting information about the temperature distribution. – for very small cross-sectional areas, space increment tiny, means time is a fraction of that so tiny increments.

- Convergence/stability issue: The time step must be a given fraction of the step size to reach a stable solution. This means that if we want a highly precise mesh grid, we also require an even larger amount of time steps which can be very inefficient and unnecessary for the computer. In our code we have included a stability checking function, which tells if the code would reach a stable solution or not. This was set using the variable d, defined earlier where $d\le0.25$ for stability.

- Alternative approach: To solve the convergence issues, we could have used an implicit method to solve our PDE such as the Crank-Nicholson method. However, this was not chosen, as it is far more inefficient computationally.